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Limiting angular dependencies of heat transfer and stratification in a heat-generating fluid

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Abstract

A theoretical study is carried out for the distribution of heat flux to the boundary, as well as of the temperature and flow velocity in the lower part of the volume taken up by a one-component heat-generating fluid. The treatment is based on the analysis of the converging boundary layer in view of the conditions of joining its characteristics with those of the fluid in the stably stratified region of the volume. It is found that the dependence of the heat flux at the boundary, q, on the polar angle θ at $\theta_* \ll \theta \ll 1$ (where θ_* is some boundary angle), and the dependence of the temperature in the volume on the ratio of the height z to the characteristic size of the volume R, are power dependences, $q \sim \theta^a$, $T_b \sim (z/R)^b$. The exponents for the cases of laminar and turbulent boundary layers are $a = 2$, $b = 4/5$ and $a = 24/13$, $b = 9/13$, respectively. The heat flux at $\theta < \theta_*$ weakly depends on θ and assumes the minimum value at $\theta = 0$. The ratio of the minimum heat flux q_m to its average value $\langle q \rangle$, as well as the boundary angle θ_* as a function of the modified Rayleigh number, are given by the estimates $q_m/\langle q \rangle \sim Ra_I^{-1/6}$, $\theta_* \sim Ra_I^{-1/12}$. The results are in quite satisfactory agreement with experiment.

1. Introduction

The problem of melt retention in the reactor vessel during severe accidents in nuclear power plants involving the core destruction is associated with the problem of processes of heat transfer from the heat-generating fluid in a closed volume. These processes are investigated by experimental [1–4] and numerical [5–8] simulation, as well as within the framework of analytical approach [9–11]. Most of the investigation results describe the distribution of heat flux between large portions of the boundary of the volume taken up by the fluid, and the respective correlation's are, in fact, integral. At the same time, the solution of the problem of nuclear reactor safety requires the knowledge of more detailed characteristics of heat flux distribution. This is especially true for the lower portion of the boundary of the reactor vessel, where special conditions for external heat removal may occur as compared with other portions of the boundary [12] (see also Ref. [13]).

This paper describes the theoretical treatment of the distribution of heat transfer from a heat-generating fluid in the neighborhood of the lowest point (pole) of the volume taken up by the fluid. As demonstrated below, the space dependencies of the characteristics of fluid in this region are power dependencies. It is our objective to derive the exponents, while abstracting ourselves from numerical factors of the order of unity.

The subsequent section contains the general formulation of the problem. The distribution of heat transfer on the boundary is closely associated with the distribution of temperature and flow velocity of fluid in the

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Nomenclature

a, b, f, d	exponents in Eq. (22)
BL	boundary layer
c	specific heat of fluid
g	acceleration due to gravity
Q	volumetric heat release
q	heat flux
$\langle q \rangle$	average value of heat flux
q_{m}	minimum value of heat flux
R	curvature radius of pool boundary at the pole
Ra_I	modified Rayleigh number, $Ra_I = \frac{\alpha g Q R^5}{\nu \chi \lambda}$
T	fluid temperature in the boundary layer
T'	turbulent pulsation value of the fluid temperature in BL
T_{b}	fluid temperature in the bulk of volume
u	turbulent pulsation value of longitudinal component of velocity in BL
u'	longitudinal component of flow velocity in BL
v	transverse component of the flow velocity in BL
v'	turbulent pulsation value of transversal component of velocity in BL
$\vec{V}$	flow velocity in the bulk of volume
V_z	vertical component of the flow velocity in the bulk of volume
y	coordinate normal to the boundary
z	vertical coordinate

Greek symbols

α	thermal expansion coefficient
δ	thickness of BL
δ_{T}	thickness of thermal sublayer in turbulent BL
λ	thermal conductivity
ν	kinematic viscosity
χ	thermal diffusivity
θ	polar angle

volume, and the latter one are the subject of Section 3. Section 4 deals with the properties of converging boundary layer in the neighborhood of the pole and gives the limiting relations for the distribution of heat transfer on the boundary, as well as of the temperature and flow velocity of fluid in the bulk. Concluding remarks and comparison of the theory with experimental data are made in Section 5.

2. Formulation of the problem

We will assume that the volume, in which the fluid is enclosed, corresponds to a body of revolution around a vertical axis with the finite radius R of its curvature at the pole. We will further assume that the quantity R is, at the same time, the characteristic dimension of the entire volume in question. The position of the running point on the downward-directed portion of the boundary will be characterized by the angle between a normal to the boundary and the vertical axis, θ. The vertical coordinate, counted from the pole level, will be designated as z. In what follows, we will be interested in the values of $\theta \ll 1$ and $z \ll R$. The temperature of the boundary at $\theta \ll 1$ will be assumed constant and treated as the origin for the reference for fluid temperature. The geometry of the problem and related notations are shown diagrammatically in Fig. 1.

The heat transfer to the downward-directed portion of the boundary is determined by the characteristics of the boundary layer (BL) present in this portion (at $\theta \sim 1$, this boundary layer is similar to the well-studied [14] free-convection BL on a vertical wall in a fluid without internal heat sources). As the pole is approached at $\theta \ll 1$, the properties of the BL assume a fundamentally different behavior. Here, by virtue of the obvious geometric conditions, the BL becomes converging, this giving rise to an important requirement, which consists in that the longitudinal component of velocity in the BL, u, must vanish at the pole,

$$u(\theta = 0) = 0 \tag{1}$$

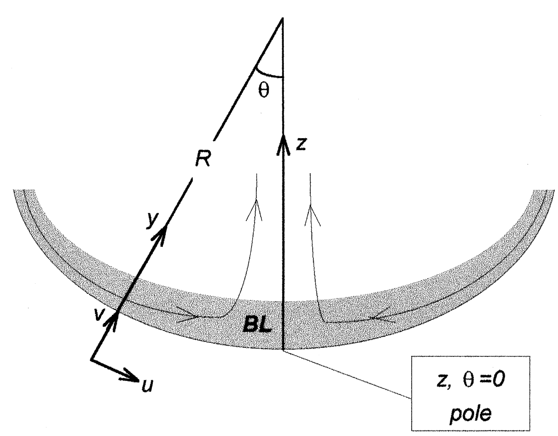

Fig. 1. Geometry of the problem.

This means that, at $\theta \ll 1$, the BL is not accelerating as in the case of $\theta \sim 1$, but decelerating. In addition, as distinct from the region of $\theta \sim 1$, where the boundary layer draws in the fluid from the bulk, at $\theta \ll 1$ it returns the fluid to that bulk. Yet another important property of the BL, acquired at $\theta \ll 1$, consists in moderation of the effect of the buoyancy force in the longitudinal direction of the BL.

Owing to condition of Eq. (1), the flow velocity and temperature of fluid in the bulk (outside of the BL) at $z \ll R$ strongly depend on z, which, in turn, has a considerable effect on the BL characteristics and, accordingly, on the distribution of the heat flux through the boundary at $\theta \ll 1$. Therefore, the problems on the BL and on the distribution of the fluid flow velocity and temperature outside of the BL must be solved simultaneously in view of the matching conditions on the free (directed into the fluid) side of the BL. These conditions consist in continuity of the fluid temperature and of the normal component of its flow velocity,

$$T = T_{\mathrm{b}} \qquad v = V_n \quad \text{at}\, y = \delta \tag{2}$$

where T and T_{b} denote the temperature in the BL and in the bulk, respectively; v and V_{n} denote the normal to the boundary component of velocity of the fluid flow in the BL and in the bulk, respectively; y is the coordinate counted from the boundary and normal to it; and δ is the boundary layer thickness.

After that, one should use the equations of hydrodynamics and heat transfer for the BL and bulk taking account of Eqs. (1) and (2) in order to derive the regularities of the behavior of heat transfer to the boundary and of the distribution of the flow velocity and temperature of the fluid in the lower part of the volume taken up by the fluid.

3. Distribution of temperature and flow velocity in the bulk

Outside of the BL, the viscosity and thermal conductivity are insignificant. The balance equation for the vertical component of momentum in the lower portion of the volume, where the flow is formed owing to the return of fluid from the BL, gives the estimate

$$T_{\mathrm{b}} - \frac{1}{\rho g \alpha}\frac{\partial p}{\partial z} \sim \frac{V_n^2}{\rho g \alpha z} \tag{3}$$

In view of relations of Eq. (2), as well as of the estimates following from the conditions of balance for mass and longitudinal component of momentum in the BL (see Section 4),

$$v \sim \frac{\delta}{\sqrt{Rz}}u \sim \sqrt{\frac{g\alpha T\delta^2}{R}} \tag{4}$$

Eq. (3) leads to the following relation valid at $z > \delta$:

$$T_{\mathrm{b}} = \frac{1}{\rho g \alpha}\frac{\partial p}{\partial z} \tag{5}$$

Further, we will estimate the characteristic magnitude of variation $\{\delta T_{\mathrm{b}}\}_{\mathrm{hor}}$ of temperature in the horizontal plane for a fixed value of the coordinate z. The respective magnitude of variation of pressure according to the balance equation for the horizontal component of momentum is related to the characteristic value of the horizontal component of velocity in the bulk by the estimate

$$\{\delta p\}_{\mathrm{hor}} \sim \rho V_{\mathrm{h}}^2 \tag{6}$$

On the other hand, the balance equation for mass in the bulk and Eqs. (2) and (4) yield

$$V_{\mathrm{h}} \sim \sqrt{\frac{g\alpha T_{\mathrm{b}}\delta^2}{z}}$$

From this estimate, in view of Eqs. (5) and (6), we derive

$$\{\delta T_{\mathrm{b}}\}_{\mathrm{hor}} \sim \left(\frac{\delta}{z}\right)^2 T_{\mathrm{b}} \tag{7}$$

Hence it follows that, at $z \gg \delta$, the temperature in the bulk depends on the vertical coordinate alone and, therefore, stable stratification takes place,

$$T_{\mathrm{b}} = T_{\mathrm{b}}(z) \tag{8}$$

The energy balance equation in the bulk has the form:

$$\left(\vec{V}\nabla T\right) = \frac{Q}{\rho c} \tag{9}$$

where $\vec{V}$ is a flow velocity in the bulk of volume, Q the volume density of heat release, c the specific heat, and ρ is the density of fluid. In view of Eq. (8), it follows from Eq. (9) that the vertical component of flow velocity V_z in the bulk, just as the temperature, is a function of the vertical coordinate alone and, in the region of $z \gg \delta$, these quantities are related by the equality:

$$T_{\mathrm{b}} = \frac{Q}{\rho c}\int_0^z \frac{\mathrm{d}z'}{V_z(z')} \tag{10}$$

4. Converging boundary layer and limiting dependencies

In analyzing the processes of heat- and mass-transfer in a converging BL, we will use the curvilinear system of coordinates described in Section 2. The cases of laminar- and turbulent-BLs will be treated separately.

4.1. Laminar boundary layer

In the range of values of the polar angle θ, which satisfy the condition

$$\frac{\delta}{R} \ll \theta^2 \ll 1 \tag{11}$$

the set of equations, expressing the property of conservation for mass, longitudinal component of momentum and energy, takes the form

$$-\frac{1}{R\theta}\frac{\partial}{\partial\theta}(\theta u) + \frac{\partial v}{\partial y} = 0 \tag{12}$$

$$-\frac{1}{R\theta}\frac{\partial}{\partial\theta}(\theta u^2) + \frac{\partial}{\partial y}(vu) - v\frac{\partial^2 u}{\partial y^2} = g\alpha(T_b - T)\theta \tag{13}$$

$$-\frac{1}{R\theta}\frac{\partial}{\partial\theta}(\theta uT) + \frac{\partial}{\partial y}(vT) = \chi\frac{\partial^2 T}{\partial y^2} \tag{14}$$

Here, ν is the kinematic viscosity, χ the thermal diffusivity, α the coefficient of volume expansion, and g is the acceleration due to gravity.

In deriving the system of equations (12)–(14), the pressure was eliminated by using the balance equation of the transverse component of momentum in the BL and Eq. (5). The validity of the left-hand part of inequality of Eq. (11) made it possible to ignore the contribution of volume heat release in Eq. (14).

The shape of transverse temperature profile in the BL is rather important from the standpoint of further derivation. In an ordinary (free-convection) BL in the vicinity of a vertical wall, with the uniform temperature of ambient media, the temperature profile is monotone [14]. A different situation takes place in the presence of stable stratification in the bulk. The temperature on the outside of the BL must coincide with the ambient temperature (see Eq. (2)), therefore, downstream of the flow, the temperature inside the BL becomes higher than that on the outside, because of convective drift from its more heated upper portions. As a result, the buoyancy force proves to be directed oppositely to the velocity of longitudinal motion of the BL, thereby ensuring its deceleration and, in the final analysis, the validity of the boundary condition of Eq. (1). Because the temperature distribution in the bulk is formed on the basis of the return of fluid from the BL, the maximum excess of temperature inside the BL above the ambient temperature must be of the same order of magnitude as the latter temperature,

$$\max(T - T_b) \sim T_b \tag{15}$$

The temperature profile in a converging BL is shown diagrammatically in Fig. 2.

Note that the nonmonotonic temperature profile in the boundary layer in the vicinity of a vertical wall with a stratified ambient medium, derived as a result of numerical calculations, was reported in Ref. [15].

We will now turn to derivation of limiting dependencies for heat transfer to the boundary and for the distribution of temperature and flow velocity in the bulk. The following three estimating relations are obtained from Eqs. (12) to (14) with account of Eq. (15):

$$v \sim \frac{\delta}{R\theta}u \tag{16}$$

$$u^2 \sim g\alpha T_b R\theta^2 \tag{17}$$

$$u\delta^2 \sim \chi R\theta \tag{18}$$

In view of the first one of conditions of Eq. (2), the space argument of T_b in Eq. (17) is assumed to be

$$z \cong R\frac{\theta^2}{2} \tag{19}$$

The following relation is derived for temperature from Eq. (10) in view of the second one of equalities from Eq. (2) and estimate (16) on condition (19):

$$T_b \sim \frac{QR^2\theta^3}{\rho cu} \tag{20}$$

One more relation follows from the very definition of heat flux,

$$q \sim \lambda\frac{T_b}{\delta} \tag{21}$$

No scale for θ is present in the range of values of the

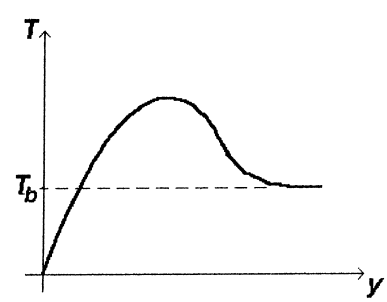

Fig. 2. Temperature profile of the converging boundary layer.

angular variable defined by inequality of Eq. (11). Therefore, the dependencies of the characteristics of fluid on θ must be power dependencies. We will define the exponents by the relations

$$q \propto \theta^a \quad T_b \propto (z/R)^b \quad u \propto \theta^f \quad \delta \propto \theta^d \tag{22}$$

The substitution of these definitions in the estimating relations of Eqs. (16)–(18) and (21) in view of Eq. (19) leads to the set of linear algebraic equations for the exponents in Eq. (22):

$$\begin{aligned} &f = b + 1 \\ &f = 1 - 2d \\ &2b + f + d = 3 \\ &a = 2b - d \end{aligned} \tag{23}$$

The solution of this system leads to the following result:

$$a = 2, \quad b = 4/5, \quad f = 9/5, \quad d = -2/5 \tag{24}$$

Relations of Eq. (22) with exponents of Eq. (24) describe the limiting dependencies for heat transfer to the boundary and for the thermohydrodynamic characteristics in the lower portion of the volume taken up by an heat-generating fluid in the case of laminar BL.

4.2. Turbulent boundary layer

As usual, we will distinguish between the average values of the components of velocity and temperature in the BL, with the designations u, v, and T, on the one hand, and the turbulent pulsation's u', v', and T', corresponding to these quantities, on the other hand. An ordinary estimate takes place for the heat flux:

$$q \sim \frac{\lambda T_b}{\delta_T} \tag{25}$$

where δ_T is the thickness of the thermal sublayer. An estimate for δ_T is obtained from two conditions on the free (directed into the fluid) side of the thermal sublayer, namely, the equality (by the order of magnitude) of the conduction and convection contributions to the heat flux to the boundary,

$$v'\delta_T \sim \chi \quad \text{at } y \sim \delta_T \tag{26}$$

and the comparability of the pulsation values of the kinetic and potential energy of fluid,

$$v'^2 \sim g\alpha T_b \delta_T \quad \text{at } y \sim \delta_T \tag{27}$$

We eliminate δ_T and v' from Eqs. (25) to (27) to derive the estimate for the heat flux,

$$q \sim \rho c \left(\chi g \alpha T_b^4\right)^{1/3} \tag{28}$$

coinciding with the case of turbulent BL on a vertical wall with isothermal ambient medium [16].

Our further objective is to determine the form of the dependence of temperature T_b in the vicinity of the boundary on the polar angle θ at $\theta \ll 1$. For this purpose, we will turn to the outer region of the turbulent BL (turbulent core), in which the viscosity and thermal conductivity are insignificant and which corresponds to $y \sim \delta$, where δ is the thickness of the turbulent BL. The condition of balance of momentum in this region leads to the following estimates:

$$v'^2 \sim g\alpha T'\delta \tag{29}$$

$$u^2 \sim g\alpha (T - T_b) R\theta^2 \tag{30}$$

The property of conservation of heat flux gives the relation

$$v'T' \sim \frac{q}{\rho c} \tag{31}$$

In the $y \sim \delta$ region, the pulsation parts of velocity and temperature are respectively comparable in magnitude with the mean transverse component of velocity and with the deviation of the average value of temperature from the value of temperature in the bulk portion adjoining the boundary layer,

$$v' \sim v, \qquad T' \sim T - T_b \tag{32}$$

We use Eqs. (16), (31) and (32) to eliminate the variables v', T' and $T - T_b$ from Eqs. (29) and (30) and derive the relations

$$\left(\frac{u\delta}{R\theta}\right)^3 \sim g\alpha \frac{q}{\rho c}\delta \tag{33}$$

$$u^3 \sim g\alpha \frac{q}{\rho c}\frac{R^2\theta^3}{\delta} \tag{34}$$

We substitute definitions of Eq. (22) in view of Eq. (19) into Eqs. (20), (28), (33) and (34) to derive the set of equations for finding the exponents defined by Eq. (22),

$$\begin{aligned} &3f + 2d - a = 3 \\ &3f + d - a = 3 \\ &2b + f + d = 3 \\ &a = \frac{8}{3}b \end{aligned} \tag{35}$$

The solution of this system is

$$a = \frac{24}{13}, \quad b = \frac{9}{13}, \quad f = \frac{21}{13}, \quad d = 0 \tag{36}$$

Eq. (22) with exponents of Eq. (36) yield the sought qualitative regularities of the behavior of thermohydrodynamic characteristics in the case of turbulent BL in the range of Eq. (11). Note that, according to Eqs. (24) and (36), exponent b determining the vertical coordinate dependence of the bulk temperature (see Eq. (22)) is less than unity. Therefore, we have the inequality:

$$\frac{\partial^2 T_\mathrm{b}}{\partial z^2} < 0 \tag{37}$$

To conclude this section, we will dwell on the case of extra small polar angles corresponding to

$$\theta^2 \ll \frac{\delta}{R} \tag{38}$$

assuming that the flow in this case is laminar. The balance equation (12) for mass yields the estimate

$$u \sim u_* \frac{\theta}{\theta_*} \tag{39}$$

where $u_* \sim v_* R\theta_*/\delta_*$, $v_* = v(\theta_*)$, and $\delta_* = \delta(\theta_*)$. The limiting angle θ_*, dividing the regions defined by inequalities of Eqs. (11) and (38), is found from the relation

$$\theta_*^2 \sim \frac{\delta(\theta_*)}{R} \tag{40}$$

In view of Eq. (39), it follows from the momentum balance equations that, in the region defined by Eq. (38), the temperature is independent of the polar angle. By virtue of Eq. (39), the first term at the left in the energy balance equation (14), on condition of Eq. (38), is to be ignored. In addition, internal heat release must be included in Eq. (14). As a result, this equation takes the form

$$v\frac{\partial T}{\partial y} = \chi \frac{\partial^2 T}{\partial y^2} + \frac{Q}{c\rho} \tag{41}$$

Hence it follows that in the range of angles $\theta \ll \theta_*$ the boundary layer thickness, just as the temperature in the layer, is independent of the angle θ, and these quantities have the estimates

$$\delta \sim \delta_* \sim \chi/v_*, \qquad T \sim T(\theta_*) \sim \frac{Q\delta_*^2}{\lambda} \tag{42}$$

It follows from Eqs. (42) and (21) that, at $\theta \ll \theta_*$, the heat flux to the boundary is nearly constant. At $\theta = 0$ it corresponds to the minimum which, in view of Eq. (40), is given by the estimate

$$q_\mathrm{m} \sim q_* \equiv q(\theta_*) \sim \langle q \rangle \theta_*^2 \tag{43}$$

where $\langle q \rangle \sim QR$ is the average value of the flux density over the entire boundary.

5. Discussion and conclusion

The main implications of the foregoing analysis are as follows. For small values of the polar angle $\theta \ll 1$, the heat flux at the boundary sharply decreases with the angle, due to the broadening of the converging BL and temperature stratification in the bulk. A concentration of horizontal isotherms in the bulk volume takes place as a consequences of the inequality Eq. (37). The asymptotic behavior of the heat flux and the bulk temperature at $\theta \ll 1$ and $z \ll R$ depends on a fluid flow regime in the converging BL.

In the case of the laminar converging BL, the asymptotic dependencies at $\theta_* \ll \theta \ll 1$ and $\delta \ll z \ll R$, according to Eqs. (22) and (24), are determined by the expressions:

$$q \propto \theta^2 \qquad T_\mathrm{b} \propto \left(\frac{z}{R}\right)^{4/5} \tag{44}$$

At $\theta \sim \theta_*$, where the angle $\theta_* \ll 1$ is defined by Eq. (40), the decrease of the heat flux slows down and, at $\theta \ll \theta_*$, it becomes nearly constant taking a minimum value at $\theta = 0$.

The asymptotic dependencies for the case of turbulent regime in the converging boundary layer, according to Eqs. (22) and (36) have the form:

$$q \propto \theta^{24/13} \qquad T_\mathrm{b} \propto \left(\frac{z}{R}\right)^{9/13} \tag{45}$$

A converging turbulent boundary layer takes place when the turbulent mode could develop at an earlier, upstream stage of the BL, at $\theta \sim 1$. In accordance with Eq. (36), in view of definitions of Eq. (22), the Reynolds number at $\theta \ll 1$ varies by the law

$$Re \equiv (u\delta/v) \propto \theta^{21/13} \tag{46}$$

according to which, in the case of relatively not large excess over the threshold of transition to turbulence, which is important from the standpoint of development of safe nuclear reactor (for values of modified Rayleigh number of $Ra_I \leq 10^{17}$), the Reynolds number drops fairly rapidly below the critical value as the polar angle decreases, this leading to the transition of converging BL to the laminar mode, and then we have the asymptotic Eq. (44) again.

We will now estimate the ratio of the minimum value of heat flux at $\theta = 0$ to the average value of the flux on the boundary, as well as the value of the boundary angle θ_*, behind which (at $\theta \ll \theta_*$) the heat

Table 1
Comparison of the theory with experimental data on the heat flux distribution

Experiment	Data	Theory
Frantz and Dhir ([17], UCLA facility $Ra_I = (3 \div 8) \times 10^{13}$	$(q_{\min}/\langle q\rangle) \approx 0.1$, $(q(\theta)/\langle q\rangle) = a + b\theta^2$	$(q_{\min}/\langle q\rangle) \sim 10^{-2}(q(\theta)/\langle q\rangle) \approx a + b\theta^2$
Kymäläinen et al. [18], COPO facility $Ra_I \sim 10^{14} \div 10^{15}$	$(q_{\min}/\langle q\rangle) \approx 0$	$(q_{\min}/\langle q\rangle) \sim 10^{-3}$
Bernaz et al. [19], BALI facility $Ra_I \sim 10^{15} \div 10^{17}$	$(q_{\min}/\langle q\rangle) < 10^{-2}$	$(q_{\min}\langle q\rangle) \sim (5 \div 1) \times 10^{-3}$

flux remains virtually constant. According to Eqs. (22) and (24), the behavior of the BL thickness at $\theta \ll 1$ is defined by the formula

$$\delta(\theta) = \delta_0 \theta^{-2/5} \tag{47}$$

where δ_0 is of the order of the BL thickness at $\theta \sim 1$. The substitution of Eq. (47) into Eq. (40) gives

$$\theta_* \sim (\delta_0/R)^{5/12} \tag{48}$$

In accordance with definition of Eq. (21) and the results of [9,10], the dependence of δ_0 on the modified Rayleigh number is given by the estimate $\delta_0/R \sim Ra_I^{-\gamma_{dn}}$, where $\gamma_{dn} \cong 0.2$. Making use of this estimate and Eqs. (40), (43) and (48), we get estimating relations for the ratio of the minimum heat flux on the boundary to its average value and for the boundary angle θ_*:

$$(q_{\mathrm{m}}/\langle q\rangle) \sim Ra_I^{-1/6} \qquad \theta_* \sim Ra_I^{-1/12} \tag{49}$$

In conclusion, we will dwell on the comparison of our results with experimental data. All the known experiments on the subject in question demonstrate a sharp decrease of the heat flux with polar angle at $\theta \ll 1$ and temperature stratification in the bulk region of the volume with a heat generating fluid at $z \ll R$, as well. Comparison of theoretical results for the heat flux distribution described by Eqs. (44) and (49) with experimental data obtained in Refs. [17–19] is given in Table 1. There is a qualitative agreement between theory and experiment on the heat flux angular dependency and the minimum to average heat flux ratio. However, to perform a more detailed comparison, a higher precision and space resolution for the heat flux distribution measurements are probably required.

In [20], the temperature distribution in a volumetrically heated fluid was registered by optical methods. In Fig. 3 shown is a typical hologram from [20] describing the distribution of isotherms. A distinctive picture of

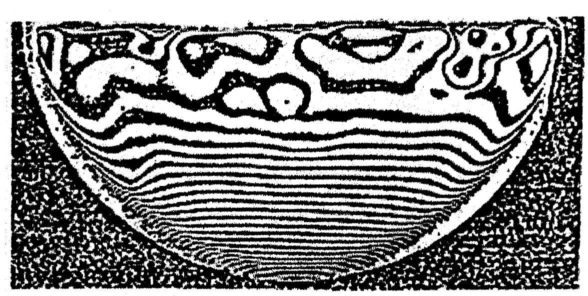

Fig. 3. Typical hologram from [20] for the distribution of isotherms.

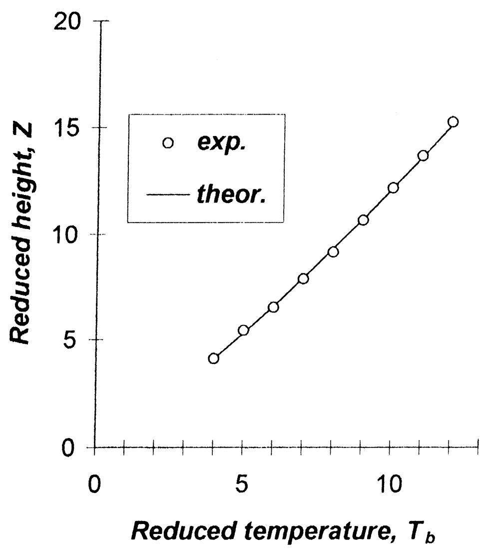

Fig. 4. Quantitative comparison of the theory with experimental data from [20] on the fluid temperature distribution.

stable temperature stratification in the bulk region of the lower portion of the fluid volume is seen at this hologram. In addition, these data are indicative of the concentration of horizontal isotherms in downward direction. Such behavior is in agreement with the theoretical inequality (37). Fig. 4 shows a quantitative comparison between the theory and experimental data on the bulk temperature distribution obtained in [20] at the power heat release corresponding to $Ra_I = 1.04 \times 10^8$. A theoretical dependency of the temperature on a dimensionless reduced height, in accordance with Eq. (44), was taken in the form $T_b \sim Z^{4/5}$. There is close correspondence between the theory and the experimental data with discrepancy at most 2.7%. Therefore, on the whole, one can talk about quite satisfactory agreement between theory and experiment.

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